# Analysis and Design of Microwave Linearized Amplifiers Using Active Feedback

EDUARDO BALLESTEROS, FÉLIX PÉREZ, AND JORGE PÉREZ

*Abstract*—**Intermodulation distortion of active feedback amplifier systems has been analyzed using a simple method. Starting from the amplitude characteristic of both amplifiers—the main and the auxiliary—it is possible to calculate the intermodulation distortion of the complete system. Analytical results show that third-order intermodulation products can be considerably reduced when an active feedback with correct amplitude and phase is employed.**

**Experimental measurements have been made on a one-stage bipolar transistor amplifier that confirm the theoretical analysis.**

## I. Introduction

In certain microwave applications there is a need for amplifiers capable of delivering high output power levels with a low intermodulation distortion. Such is the case in, e.g., multicarrier telecommunications systems, amplitude-modulated transmitting systems (TV broadcasting), and digital radio systems. The conventional approach for achieving this result is to use amplifiers operating at an output level far below their real capabilities (back-off approach). However, this procedure results in very low efficiency amplifier systems that require considerably oversized active devices, which makes this approach impractical in many cases. As a result, for more than a decade, alternative configurations have been investigated in order to achieve the design goals. Among these are the reinjection amplifier systems [1], the passive feedback systems [2], the predistortion linearized amplifiers [3], active device bias control systems [4], and low-frequency feedback systems [5].

The nonlinear behavior analysis of this type of structure is usually implemented by simple approximations which, frequently, allow only insufficient predictions of the signal/intermodulation ratio of the system in relation to the output power. On the other hand, the literature also contains sophisticated analysis procedures of the nonlinear distortion introduced in the narrow-band modulated signals by the microwave amplifiers [6].

In this paper a new linearization configuration of microwave power amplifiers using active feedback networks is presented. Because of its structural characteristics, it can be easily compared with the amplifier systems that operate by the reinjection principles or passive feedback, although their operating principles are, in a certain way, more similar to those of the predistorted systems.

The analysis of the structure is conducted by a procedure which tries to combine accuracy and simplicity, and a simplification of the mentioned nonlinear distortion analysis methods can be considered.

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## II. Linearization of Active Feedback Amplifiers: Basic Concepts

Fig. 1 shows the basic diagram of an amplifier system which uses the active feedback principle. It consists of an input coupler with coupling $C_i$, a main amplifier with small-signal gain $G_m$ and output power at 1 dB compression $P_{1dBm}$, an output coupler with coupling $C_o$, and, lastly, an auxiliary amplifier characterized by the parameters $G_a$ and $P_{dBa}$.

In a first modeling of this system, the following hypotheses will be assumed:

a) Each of the elements of the system has all its ports perfectly matched, independently of the signal level incident on them.

b) The inverse transmission of the amplifiers is negligible.

c) The amplifiers are narrow-band. For purposes of their nonlinear behavior in the band, the zero-memory system can be considered. This condition implies that the AM–PM conversion of the amplifiers is not taken into account, and its study is out of the scope of this model.

d) It will be allowed that the amplifiers can be characterized by their amplitude characteristic functions (or describing functions). This assumption is justified, among others, by two reasons. The first is that the microwave active devices—especially the encapsulated ones—tend to remove the harmonic contents of the device accessible port signals. The second is that all the feedback systems show low-pass-type responses in order to avoid oscillations at high frequencies.

e) At operating frequency, the negative-feedback condition is accurately attained: that is, the loop gain has a 180° associated phase.

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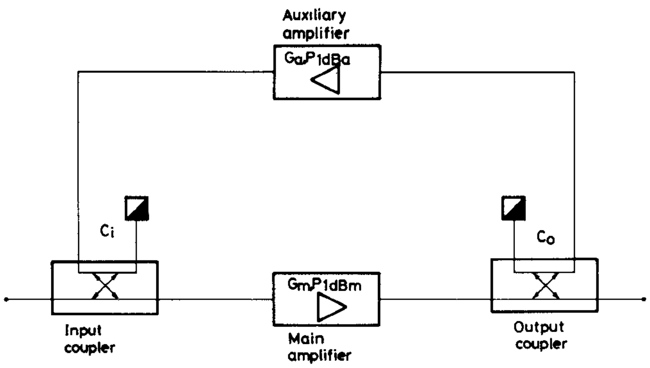



Fig. 1. Active feedback amplifier system.

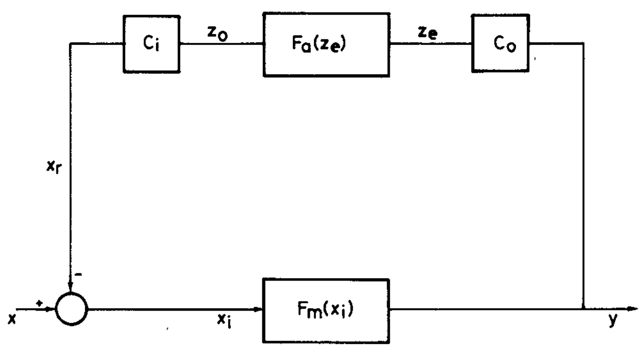



Fig. 2. Model of the amplifier system.

Under these conditions, the approximate model shown in Fig. 2 can be used to study the response of the system with monotone excitation. In this representation, the lettering indicates the signal amplitude at each point of the circuit.

The amplifiers are described by their amplitude characteristics $F_m$ (main amplifier) and $F_a$ (auxiliary amplifier).

The amplitude characteristic of the open-loop system shown in Fig. 2 will be

$$x_r = F_1(x_i) = C_i \cdot F_a(C_o \cdot F_m(x_i)) \tag{1}$$

and the amplitude characteristic of the closed-loop system will be given by

$$y = F_t(x) = F_m(x_i) \tag{2a}$$

where $x_i$ is implicitly defined in relation to $x$ by the balance condition

$$x = x_i + x_r. \tag{2b}$$

In order to understand qualitatively the linearizing effect of active feedback, it is fitting to express the conditions (1), (2a), and (2b) in terms of the gains combined with the amplitude characteristics. In this way, taking into account that the gain associated with an amplitude characteristic function is given by

$$G(\alpha) = F(\alpha)/\alpha \tag{3}$$

where

$$\alpha = x_i \text{ (main amplifier)}$$

$$\alpha = z_e \text{ (aux. amplifier)}$$

the open loop gain will be

$$G_1(x_i) = \frac{x_r}{x_i} = C_i \cdot G_a(C_o \cdot G_m(x_i) \cdot x_i). \tag{4}$$

The gain of the complete system and the balance condition of the loop will be given, respectively, by

$$G_t(x) = y/x = \frac{G_m(x_i)}{1 + G_l(x_i)} \tag{5a}$$

$$x = x_i(1 + G_l(x_i)). \tag{5b}$$

On increasing the signal level ($x$), the main amplifier reduces its gain ($G_m(x_i)$) due to the compression effect. However, as indicated by expression (4), the feedback magnitude is also reduced ($G_l(x_i)$) not only by its own compression, but especially by that owing to the auxiliary amplifier. By adequately sizing the input and output couplers, it is possible to make the compression degree of the auxiliary amplifier suitable to compensate the gain reduction from compression of the main amplifier.

It is interesting to point out that the utilization of an active feedback network allows linearizing the amplitude characteristic of the main amplifier with two important advantages in relation to passive feedback:

a) The gain loss of the main amplifier can be very low.
b) As a direct consequence of the above, the stability of the system is not jeopardized since $G_l$ is much less than 1.

## III. Active Feedback Nonlinear Study

In this paper, the output signal/intermodulation relation ($S/I$) as a function of the output signal power will be adopted as a linearity measurement of the system. The signal power is understood as that corresponding to the output components of frequencies equal, respectively, to the two input test tones and $S/I$ relation as the difference of levels existing between any of the output third-order beating tones and the signal tones. This is justified by the fact that this is one of the typical specifications of a microwave power amplifier. Moreover, the procedure presented can be easily generalized to higher order intermodulation products.

### *A. Prediction of the S/I Relation of a System from its Characteristic Function*

The nonlinear behavior analysis of the system is implemented by starting from its voltage characteristic $v = v(u)$ (instantaneous amplitude output versus instantaneous amplitude input), which will be assumed odd since the even part of $v(u)$ does not contribute (if existing) to the narrow-band response. This starting point does not constitute a limitation of the procedure since, as Blachman has demonstrated [7], the odd part of $v(u)$ can be determined starting from the amplitude characteristic $F(a)$.

The relation which makes it possible to determine the in-band amplitude characteristic as a function of the voltage characteristic is now classic [7]:

$$F(a) = \frac{2}{\pi}\int_0^{\pi} v(a\cdot\cos\theta)\cdot\cos\theta\, d\theta$$

or ($\theta = \pi/2 - \theta$)

$$F(a) = \frac{2}{\pi}\int_0^{\pi} v(a\cdot\sin\theta)\cdot\sin\theta\, d\theta. \quad (6)$$

The response of the system to the two-tone test signal can be calculated with the following procedure [8]. Let the input signal be

$$x(t) = a\cdot(\sin\omega_1 t + \sin\omega_2 t) = 2a\cdot\sin\omega_0 t\cdot\cos\omega_d t \quad (7)$$

where

$$\omega_0 = (\omega_1 + \omega_2)/2 \text{ and } \omega_d = (\omega_1 - \omega_2)/2 \qquad (\omega_d \ll \omega_0). \quad (8)$$

The output signal is then given by

$$y(t) = v(2\cdot a\cdot\sin(\omega_0 t)\cdot\cos(\omega_d t)). \quad (9)$$

Generally $\omega_0$ and $\omega_d$ are incommensurable so that $y(t)$ is not periodic and does not allow a straightforward Fourier series expansion. As a result, we use the auxiliary bidimensional signal:

$$\hat{y}(\zeta_0,\zeta_d) = F(2\cdot a\cdot\sin(\omega_0\cdot\zeta_0)\cdot\cos(\omega_d\cdot\zeta_d)) \quad (10)$$

which is periodic in $\zeta_0$ and in $\zeta_d$, having odd symmetry in relation to its first argument and even in relation to the second one. Then, $\hat{y}(\zeta_0,\zeta_d)$ admits a double series expansion of the type

$$\hat{y}(\zeta_0,\zeta_d) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} C_{nm}(a)\cdot\sin(n\omega_0\cdot\zeta_0)\cdot\cos(m\omega_d\cdot\zeta_d) \quad (11)$$

where

$$C_{nm}(a) = \frac{4}{T_0 T_d}\int_{-T_0/2}^{T_0/2}\int_{-T_d/2}^{T_d/2} v(2a\cdot\sin(\omega_0\cdot\zeta_0)\cdot\cos(\omega_d\cdot\zeta d)) \cdot\sin(n\omega_0\cdot\zeta_0)\cdot\cos(m\omega_d\cdot\zeta_d)\cdot d\zeta_0\cdot d\zeta_d. \quad (12)$$

The expression (12) can be simplified using the angular variables $\theta = \omega_0\zeta_0$, $\phi = \omega_d\zeta_d$, resulting in

$$C_{nm}(a) = \frac{4}{\pi^2}\int_0^{\pi}\int_0^{\pi} v(2a\cdot\sin\theta\cdot\cos\phi)\cdot\sin(n\theta) \cdot\cos(m\phi)\cdot d\theta\cdot d\phi. \quad (13)$$

Now taking into account that $y(t) = \hat{y}(t,t)$, the amplitude of each of the two components of signal and third-order intermodulation can be obtained:

$$S(a) = \frac{1}{2}C_{11}(a) = \frac{2}{\pi^2}\int_0^{\pi}\int_0^{\pi} v(2a\cdot\sin\theta\cdot\cos\phi)\sin\theta \cdot\cos\phi\cdot d\theta\cdot d\phi \quad (14)$$

$$I_3(a) = \frac{1}{2}C_{13}(a) = \frac{2}{\pi^2}\int_0^{\pi}\int_0^{\pi} v(2a\cdot\sin\theta\cdot\cos\phi)\sin\theta \cdot\cos 3\phi\cdot d\theta\cdot d\phi. \quad (15)$$

Considering lastly the relation (6), $S(a)$ and $I_3(a)$, hereafter referred to as $I(a)$, can be expressed as a function of $F(a)$:

$$S(a) = \frac{1}{\pi}\int_0^{\pi} F(2a\cdot\cos\phi)\cdot\cos\phi\cdot d\phi \quad (16)$$

$$I(a) = \frac{1}{\pi}\int_0^{\pi} F(2a\cdot\cos\phi)\cdot\cos 3\phi\cdot d\phi. \quad (17)$$

In the previous system $F(a)$ can be eliminated, thus obtaining a direct relation between $S(a)$ and $I(a)$:

$$I(a) = S(a) - 4\int_0^1 \alpha^2\cdot S(a\alpha)\cdot d\alpha. \quad (18)$$

Finally, the relation $S/I$ can be expressed as a function of the amplitude characteristic:

$$S/I = \frac{\int_0^{\pi} F(2a\cdot\cos\phi)\cos\phi\cdot d\phi}{\int_0^{\pi} F(2a\cdot\cos\phi)\cos 3\phi\cdot d\phi}. \quad (19)$$

In our case, as will be shown later, $F(a)$ is determined in the form of a table of discrete values. In evaluating the expression (19) we have used two alternative methods:

*1) Polynomial Interpolation:* It is possible to approximate $F(\alpha)$ by fitting a table of values with a least-squares method:

$$F(\alpha) = A_1\alpha + A_3\alpha^3 + A_5\alpha^5 + \cdots A_{2n+1}\alpha^{2n+1}\cdots + A_{2p+1}\alpha^{2p+1}. \quad (20)$$

Then, the use of the Chebysheve transform leads immediately to

$$S(a)/I(a) = \frac{\sum_{n=0}^{2p+1} A_{2n+1}\binom{2n+1}{n} a^{2n+1}}{\sum_{n=1}^{2p+1} A_{2i+1}\binom{2n+1}{n-1} a^{2n+1}} \quad (21a)$$

where

$$\binom{i}{j} = \frac{i(i-1)(i-2)(i-j+1)}{j(j-1)(j-2)\cdots 1}. \quad (21b)$$

This last expression allows a greater simplicity in evaluating the $S/I$ relation. The drawback of this method is that the errors in the approximation of $F(\alpha)$ translate into errors in the determination of the $S/I$ relation difficult to predict. Its accuracy will diminish as the amplifier operates in very large signal.

*2) Fourier Series Expansion of v(u):* In this case, we start from the Fourier series expansion of the voltage characteristic:

$$v(u) = \sum_{n=1}^{\infty} a_n\cdot\sin\left|\frac{n\pi}{X}u\right|, \qquad |u| < X \quad (22)$$

which, by common knowledge, results in the expansion [7]

$$F(a) = 2\sum_{n=1}^{\infty} B_n J_1\left(\frac{n\pi a}{X}\right). \quad (23)$$

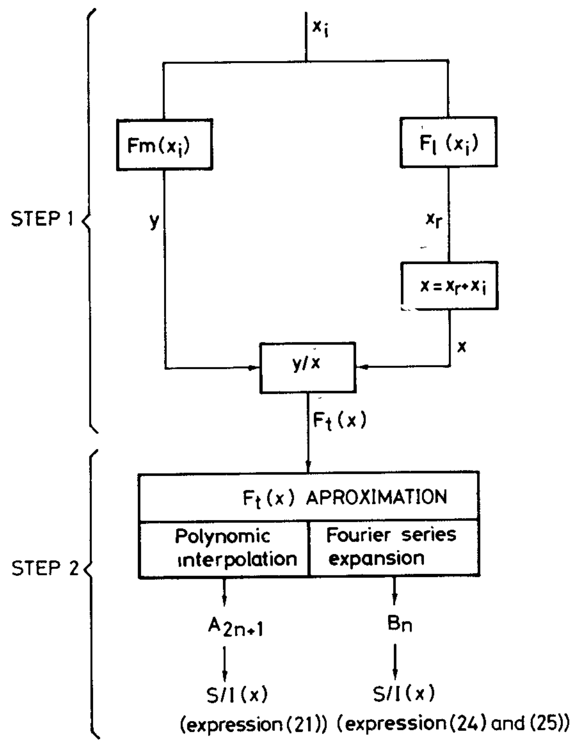



Fig. 3. Chart of $S/I$ calculation method.

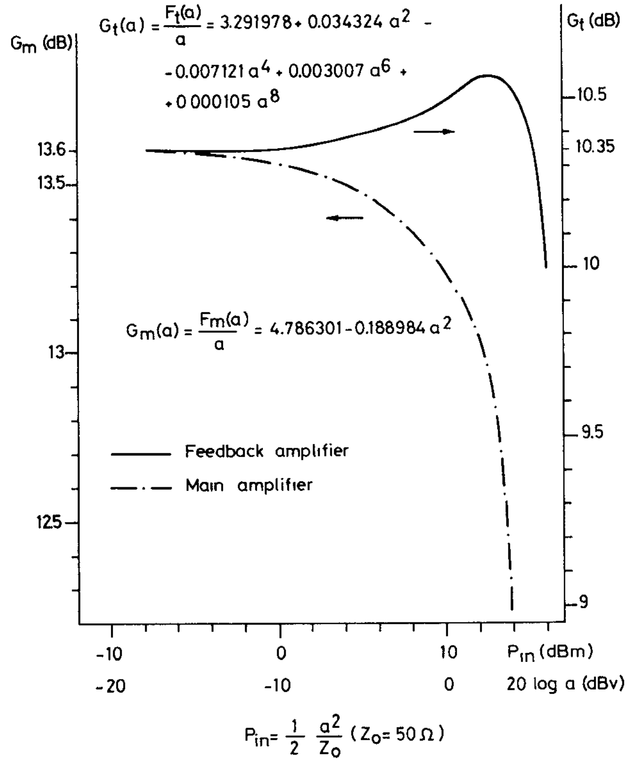



Fig. 4. Amplitude characteristic of the active feedback amplifier

Applying the Chebyshev transformation:

$$S(a) = 2\sum_{n=1}^{\infty} B_n J_0\left(\frac{n\pi a}{X}\right) J_1\left(\frac{n\pi a}{X}\right) \tag{24}$$

$$I(a) = 2\sum_{n=1}^{\infty} B_n J_1\left(\frac{n\pi a}{X}\right) J_2\left(\frac{n\pi a}{X}\right) \tag{25}$$

where the coefficients $b_n$ are given by

$$B_n = -\frac{n\pi}{X^2}\int_0^{\pi/2}\int_0^{X} xF(x\sin\phi)\cos\left(\frac{n\pi}{X}x\right) dx\cdot d\phi \tag{26}$$

as demonstrated with the help of expression (6).

### B. Application to Active Feedback

In calculating the $S/I$ relation of the active feedback amplifier, we will follow the routine shown in the chart of Fig. 3.

The first step will be to find the amplitude characteristic of the complete system, starting from the amplitude characteristics of the amplifiers ($F_m$ and $F_a$) and from the coupling values of the couplers ($C_o$ and $C_i$) (see Fig. 2).

By means of expressions (2a), (1), and (2b) for a set of discrete amplitude values at the main amplifier input, $x_i$, a correspondence table can be found between the signal amplitudes at the input and output of the complete system $(y - x)$, which define the amplitude characteristic of the complete system ($F_t(x)$) for a discrete set of values of $x$.

We can later use any of the proposed methods to evaluate the $S/I$. This can be done by polynomial interpolation, by the expressions (20) and (21), or by the Fourier expansion of $F_t(x)$, with the help of expressions (24), (25), and (26).

## IV. Design of an $L$-Band Linearized Amplifier Stage [9]

In order to verify the validity of the ideas mentioned in the previous sections, an $L$-band active feedback linearized amplifier has been designed. As active devices, the bipolar transistors BFT-98 (main amplifier) and BFR-91 (auxiliary amplifier) have been chosen for simplicity. The amplifiers were designed employing small-signal conventional procedures, with the single particularity of selecting the matching networks which can introduce a minimum transmission phase shift to improve the bandwidth.

Once the amplifiers had been made, their power outputs were experimentally characterized at 1 dB compression, obtaining 27 dBm for the main amplifier and 19 dBm for the auxiliary amplifier. The small-signal gains were 13.6 and 10 dB, respectively.

The amplitude characteristics of the amplifiers were later modeled by means of cubic polynomials without even terms. The coefficients of the polynomial were determined in such a way that the small-signal gain and the 1 dB compression output power were suitable in each case:

$$F_m(x) = 4.78630092x - 0.188984209x^3 \tag{27}$$

$$F_a(x) = 3.16227766x - 0.343894728x^3. \tag{28}$$

Using the method of analysis described in the previous section, the value of the couplings of the input and output couplers was optimized so that the output power of the system with an $S/I$ ratio of 40 dB would be maximum: $C_i = 20.53$ dB and $C_o = 11.21$ dB, which implies a small-signal gain loss of 3.25 dB. The amplitude characteristic of

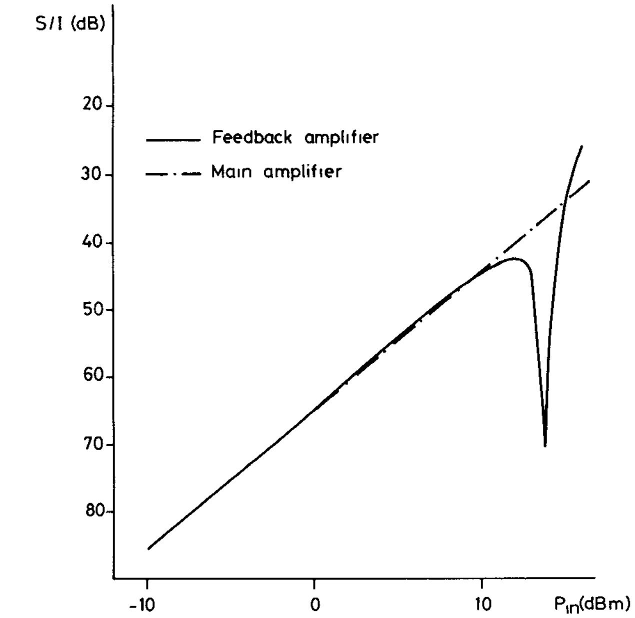
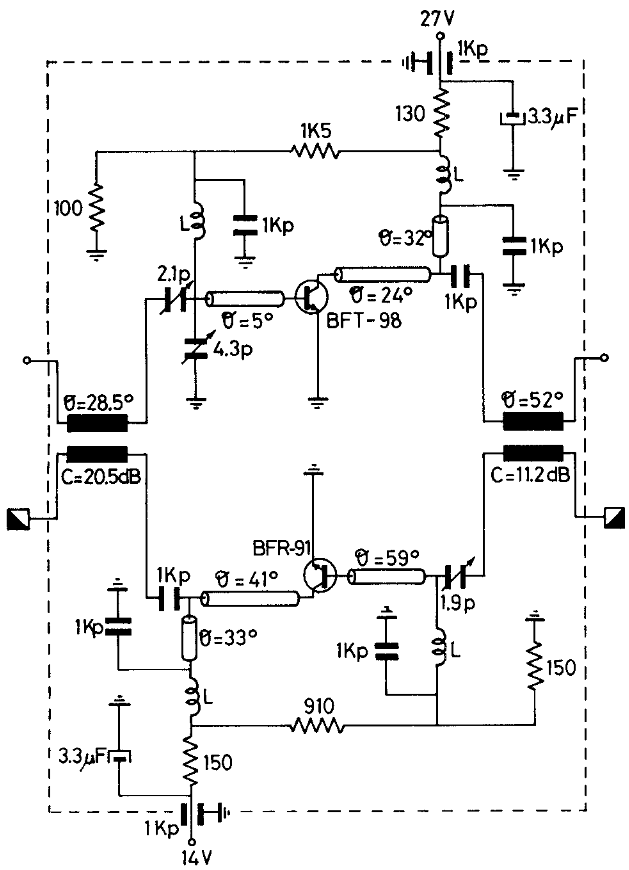



Fig. 5. $S/I$ of the active feedback amplifier.




Transmision lines: Z = 50Ω; electrical length at 1 GHz

Fig. 6. $L$-band linearized amplifier stage.

the complete system, as well as the polynomic approximation obtained, is shown in Fig. 4.

The resultant $S/I$ response is similar for the two calculation methods related and is shown in Fig. 5.

Finally, the physical designs of the input and output couplers were implemented with the requirements of obtaining the optimum couplings and a loop gain with 180° associated phase. Thus, we obtained the amplifier system shown in Fig. 6. The implementation of the matching and polarization networks was made on a plastic substrate of low dielectric constant (CuClad), whereas the couplers were made on a plastic substrate of high dielectric constant (Epsilam-10) in order to reduce their physical length. The circuit was assembled; its measured small-signal parameters indicate an adequate adaptation (COE < 2) in a 130 MHz band centered at 1 GHz, with a gain of 10.5 dB. The two-tone response is the one shown in Fig. 7, and it successfully agrees with the one projected theoretically, showing a 3.2 dB increase in output power with $S/I$ of 40 dB in relation to the amplifier without feedback. The only drawback of the structure is its sensitivity to the temperature, since the gain changes of the active devices impair the linearization effect. In some applications a gain control system as a function of the temperature must be introduced.

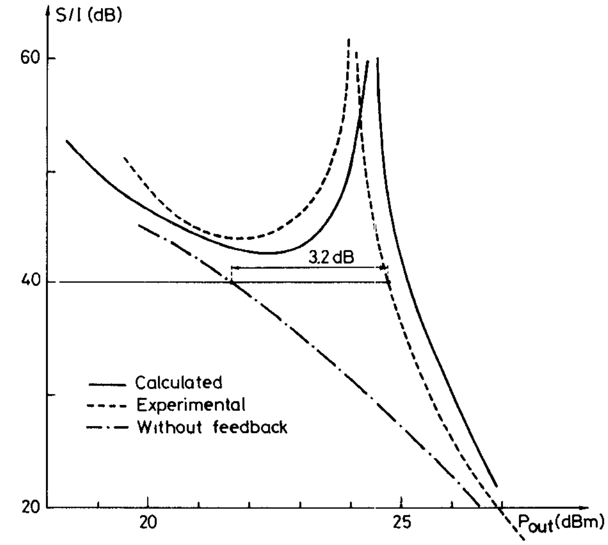



Fig. 7. Experimental results.

## V. Conclusions

The linearization method of active feedback amplifiers allows considerable improvement in power output with a given $S/I$ relation without important gain losses. An $L$-band amplifier system has been designed which makes it possible to double the output power with an $S/I$ of 40 dB and increases the efficiency by 55 percent in relation to the main amplifier. The method shows advantages over the feedforward systems with regard to the simplicity of the structure and the nature of the auxiliary amplifier, which can have a saturation power much less than the main one.

The analysis method presented has been proved to be a useful design tool for this type of linearization system, and is capable of being extended to other structures such as those of reinjection or predistortion.

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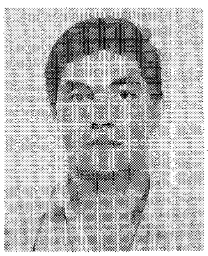

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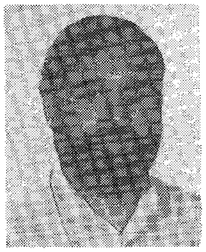

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